

LTSE 2019 (India) Solutions

LEAD Trust

March 3, 2019

1 Physics

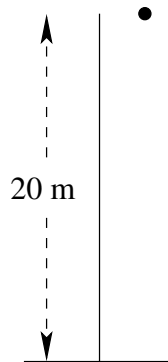
1. The measure of motion of a body is

Answer:- (c)

By definition, momentum of a body is the amount of motion possessed by it.

2. A ball is dropped gently from a building 20m high. The velocity with which it strikes the ground is v and time taken to strike is T then (v, T) is (assume acceleration due to gravity = $10m/s^2$)

Answer:- (c)



Given,

initial velocity, $u = 0$;

acceleration due to gravity, $g = 10ms^{-2}$

distance travelled, $S = 20m$

Applying equation of motion

- (a) For final velocity

$$v^2 - u^2 = 2aS$$

$$v^2 - 0 = 2(10)(20)$$

$$v = \sqrt{400}$$

$$v = 20ms^{-1}$$

- (b) For time taken

$$v = u + aT$$

$$20 = 0 + (10)T$$

$$T = \frac{20}{10}$$

$$T = 2s$$

Hence $(v, T) = (20, 2)$

3. A bullet of 20g is horizontally fired with 150ms⁻¹ from a pistol of mass 2kg. The recoil kinetic energy of the pistol is

Answer:- (b)

Given,

$$\text{mass of the bullet} = m_1 = 20g = 0.02kg$$

$$\text{initial velocity of bullet} = u_1 = 0$$

$$\text{final velocity of bullet} = v_1 = 150ms^{-1}$$

$$\text{mass of the pistol} = m_2 = 2kg$$

$$\text{initial velocity of pistol} = u_2 = 0$$

$$\text{final velocity of bullet} = v_2 = ?$$

Since no net external force is acting on the system, momentum of (bullet + pistol) system remains constant.

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$$(0.02)(150) + (2)(v_2) = 0 + 0$$

$$v_2 = \frac{-3}{2} \quad v_2 = -1.5ms^{-1}$$

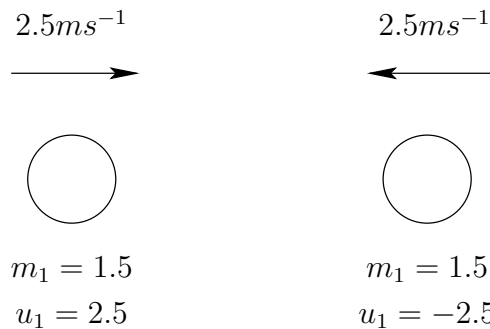
Kinetic energy

$$K = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2)\left(\frac{-3}{2}\right)\left(\frac{-3}{2}\right)$$

$$K = \frac{9}{4} = 2.25J$$

4. Two objects each of mass 1.5kg are moving along same line towards each other with speeds 2.5ms⁻¹ each before they collide. After the collision they stick to each other. The velocity of combined system just after the collision is

Answer:- (d)



$$\text{Momentum before collision} = m_1u_1 + m_2u_2 = (1.5)(2.5) + (1.5)(-2.5) = 0$$

Hence after collision momentum remains same since external force on the system of 2 bodies remains zero. (**Note:** Forces during the collision between the two bodies are all internal.)

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$0 = (m_1 + m_2)v$$

$$v = 0$$

5. Distance between the two masses is reduced to half. The % change in their mutual gravitational force is

Answer:- (b)

$$F = \frac{Gm_1m_2}{r^2}$$

If distance is reduced by half

$$r \rightarrow \frac{r}{2}$$

$$F' = \frac{Gm_1m_2}{\left(\frac{r}{2}\right)^2} = 4 \frac{Gm_1m_2}{r^2} = 4F$$

$$\text{Change } \Delta F = (4F - F) = 3F$$

$$\% \text{ Change} = \frac{\Delta F}{F} \times 100 = \frac{3F}{F} \times 100 = 300 \% \text{ increase}$$

6. A lens has a power of -2.5D. The focal length and type of lens is

Answer:- (a)

Power of lens,

$$P = \left(\frac{1}{f}\right)$$

$$-2.5 = \frac{1}{f}$$

$$f = -0.4m$$

Since focal length is negative, it is a concave lens.

7. Concave lens has focal length of 15cm. The image is formed at 10 cm from the lens. The object distance is

Answer:- (b)

Given, $f = -15cm$ (Negative sign taken for concave lens)

$$v = -10cm$$

For a lens:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{-1}{10} - \frac{1}{u} = \frac{-1}{15}$$

$$\frac{1}{u} = \frac{1}{15} - \frac{1}{10}$$

$$u = -30cm \text{ from lens}$$

8. Two lamps rated 100W at 220V and 60W at 220V each are connected in parallel across a 220V power supply. The current drawn from the supply is

Answer:- (b)

lamp 1: 100W; 220V

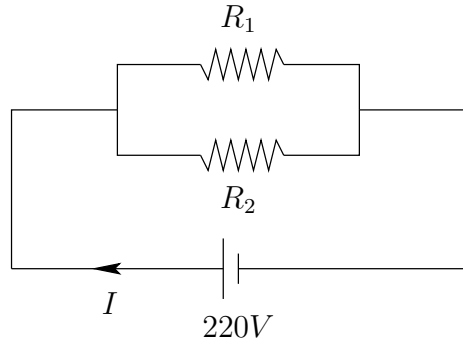
lamp 2: 60W; 220V

Resistance

$$R = \frac{V^2}{P}$$

$$R_1 = \frac{(220)^2}{100}$$

$$R_2 = \frac{(220)^2}{60}$$



$$\text{Net } R_{eff} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(220)^2 (220)^2}{\frac{100}{60} + \frac{60}{60}} = \frac{(220)^2}{160}$$

$$I = \frac{V}{R_{eff}} = \frac{220}{\frac{(220)^2}{160}} = \frac{8}{11} A$$

9. A wire of resistance R is cut into 5 parts and the parts are connected in parallel to each other. The effective resistance of the resultant combination is

Answer:- (d)

Wire of resistance, R , is cut in 5 pieces of equal length.

Resistance of each = $\frac{R}{5}$ (Since $R \propto \text{length}$)

Parallel combination of 5 resistors,

$$\frac{1}{R_{eff}} = \frac{5}{R} + \frac{5}{R} + \frac{5}{R} + \frac{5}{R} + \frac{5}{R}$$

$$R_{eff} = \frac{R}{25}$$

10. At the time of short circuit, the current in the circuit

Answer:- (c)

At the time of short circuit, the wire of very small resistance is connected between the points of different potentials.

According to Ohm's law:

$$\frac{V}{R} = I$$

Since R decreases while V remaining same, I will increase.

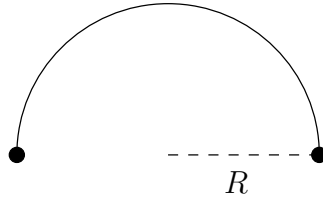
11. A circular coil of copper wire is rotated in a uniform magnetic field. The direction of induced current changes once in each

Answer:- (b)

According to Lenz's law, the induced current is produced in order to oppose any change in flux through the loop in this system, the change in the direction of current (clockwise or anticlockwise) occurs when coil is perpendicular to the plane of loop which happens twice for every rotation.

12. A particle moving along circular path reaches a diametrically opposite point. The ratio of its average speed to the magnitude of average velocity is

Answer:- (c)



distance covered = πR
 displacement = $2R$

$$\text{Avg Speed} = \frac{\text{distance}}{\text{time}} = \frac{\pi R}{t}$$

$$\text{Avg velocity} = \frac{\text{displacement}}{\text{time}} = \frac{2R}{t}$$

$$\frac{\text{Avg Speed}}{\text{Avg velocity}} = \frac{\frac{\pi R}{t}}{\frac{2R}{t}} = \frac{\pi}{2}$$

13. A loop whose plane lies in xy plane (as shown in figure) is rotated by a small angle in clockwise sense as seen from positive y axis. If a uniform magnetic field exist in the direction towards negative z axis then the direction of induced current in the loop as viewed from positive z axis is

Answer:- (a)

Lenz's law states that the direction of induced current opposes any change in flux in the given system, the flux through loop decreases, hence induced current must be produced so that it produces flux in the same direction as produced by the magnetic field.

According to right hand thumb rule the induced current must be in clockwise sense as viewed from positive z axis in order to support flux.

14. If two pieces of different metals completely immersed in water experiences equal up thrust, then

Answer:- (c)

Archimedes principle:

Upthrust/bouyant force is equal to the weight of the displaced liquid.

Since the metal pieces are fully immersed in water, their volume must be same in order to displace same amount of water.

15. Two blocks of unequal masses $m_1=4\text{kg}$ and $m_2= 8\text{kg}$ are moving with velocities $v_1=10\text{m/s}$ and $v_2=5\text{m/s}$ respectively. The net forces on m_1 and m_2 required to keep them moving with their velocities are F_1 and F_2 respectively. Then

Answer:- (d)

Force is not necessary to keep a body in motion at constant velocity.

$$F_1 = 0; F_2 = 0$$

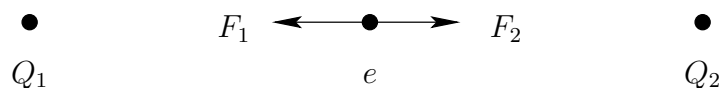
16. A person is stuck on a frictionless surface and cannot exert any horizontal force against the surface. He should get off by

Answer:- (d)

According to law of conservation of momentum, anything that the man throws will produce a recoil which pushes him. Hence he can through his spit or air that would save him. (similar to gun recoil)

17. An electron is placed in between two fixed charges of magnitude Q_1 and Q_2 are as shown. The net force on the electron is zero. Then

Answer:- (c)



Net force

$$F_1 = K \frac{Q_1 Q_2}{R^2} = (K) \frac{Q_1(e)}{9}$$

$$F_2 = K \frac{Q_1 Q_2}{R^2} = (K) \frac{Q_2(e)}{16}$$

$$F_1 = F_2 \implies \frac{K Q_1 e}{9} = \frac{K Q_2 e}{16}$$

$$16 Q_1 = 9 Q_2$$

18. A sound wave is

Answer:- (c)

Sound wave needs elastic medium to travel like gas/liquid/solid. These are longitudinal waves.

19. The temperature at which Celsius and Fahrenheit scale have equal values is

Answer:- (d)

$$\frac{T_F^\circ - 32}{180} = \frac{T_C^\circ - 0}{100}$$

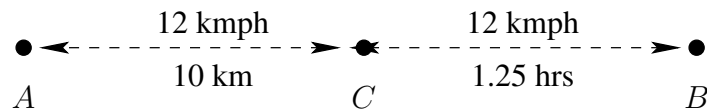
$$\frac{x - 32}{180} = \frac{x}{100}$$

$$100x - 3200 = 180x$$

$$x = -40^\circ$$

20. A person travels from A to B. if she first travels at an average speed of 12kmph for 10km, then at 8kmph for another 1.25hrs. The average speed of the entire trip is

Answer:- (c)



$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$AC = 10 \text{ km}$$

$$t_{AC} = ?$$

$$t_{AC} = \frac{\text{distance}}{\text{time}} = \frac{10 \text{ km}}{12 \text{ kmph}}$$

$$t_{AC} = 0.83 \text{ hours}$$

$$t_{CB} = 1.25 \text{ hours}$$

$$CB \implies \text{distance} = (\text{speed})(\text{time}) = 1.25 \times 8 = 10 \text{ km}$$

$$\text{Total distance} = AC + CB = 10 + 10 = 20$$

$$\text{Total time} = t_{AC} + t_{CB} = 0.83 + 1.25 = 2.08 \text{ hours}$$

$$\text{Total avg speed} = \frac{20}{2.08} = 9.6 \text{ kmph}$$

2 Logical Reasoning

41. Observe the series and fill the blank with correct number:

664, 332, 340, 170, ----, 89.

Answer:- (d)

This is an alternating division and addition series: First, divide by 2, and then add 8.

42. The sum of the ages of five children born at intervals of five years each is 100 years. What is the age of the middle child?

Answer:- (b)

Let the age of youngest child be x .

Hence,

$$(x) + (x + 5) + (x + 10) + (x + 15) + (x + 20) = 100$$

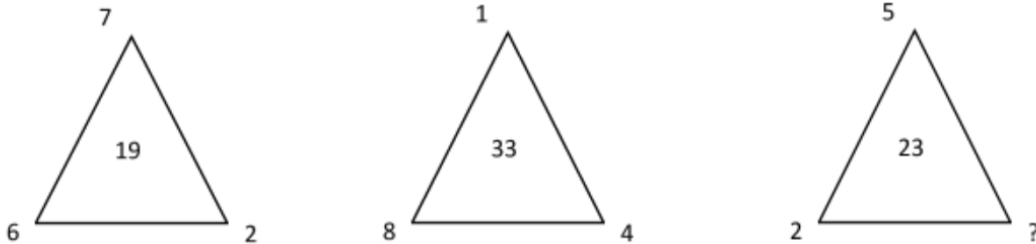
$$\text{or, } 5x + 50 = 100$$

$$\text{or, } 5x = 50$$

$$\text{or, } x = 10$$

Ages of the children are 10, 15, 20, 25 and 30. Age of middle child is 20

43. Which number replaces question mark?



Answer:- (c)

In each triangle, multiply the lower two numbers together and add the upper number to give the value in the centre. ($2 \times 9 + 5 = 23$)

44. Find next number in the series.

806, 519, 287, 232, 55, ___ .

Answer:- (c)

$$806 - 519 = 287, 519 - 287 = 232, 287 - 232 = 55, 232 - 55 = 177$$

45. Anil has a brother Deepak. Anil is the son of Prem. Vimal is Prem's father. How is Deepak related to Vimal?

Answer:- (d)

46. In a football match Prem scored more than Hisham but less than Tom. Robin scored less than Roshan but more than Tom. Whose score was the lowest in the match?

Answer:- (a)

$$\text{Roshan} > \text{Robin} > \text{Tom} > \text{Prem} > \text{Hisham}$$

47. Ram is taller than Shubham but not as tall as Deepak. Shubham is taller than Prem. Deepak is not as tall as Rohan who among them is the tallest?

Answer:- (b)

$$\text{Rohan} > \text{Deepak} > \text{Ram} > \text{Shubham} > \text{Prem}$$

48. A boy started from his home. After walking 5 km towards east, he turned to his right and walked for 8km. Then he again turned to his right and walked for 10 km. In which direction is he from his house?

Answer:- (b)

49. A 3-digit number $4a3$ is added to another 3-digit number 984 to give a 4-digit number $13b7$, which is divisible by 11 . Find a and b ?

Answer:- (d)

$413 + 984 = 1397$, which is divisible by 11 .

50. If $6xy5$ is a four digit number divisible by 55 then $(x-y)$ is equal to:

Answer:- (c)

First 4 digit starting with 6 and ending in 5 divisible by 55 is 6105 (55×111). To get further numbers divisible by 55 and ending with 5 we need to add 110 to previous number. Since we are adding 1 to both the second and third digits of the the 4 digit number. The difference of the second and third digit will remain same as the first number unless the thousands place change.

So the difference will be $(1 - 0 = 1 = x - y)$.

51. A person runs 2 km every day except on Sundays on which he runs 1 km. How many kilometre he would run by 5 th August (including), if he started on 28 th May which was a Monday?

Answer:- (d)

May - 4 days - 0 Sundays

June - 30 days - 4 Sundays

July - 31 days - 5 Sundays

August - 5 days - 1 Sunday

Total number of days = 70

Total number of Sundays = 10

Total distance covered = $70 \times 2 - 10 \times 1 = 130$

52. The acute angle between the minute hand and the hour hand of a clock, when the time is 4.20 AM, is:

Answer:- (c)

Hour hand covers 360 degrees in 12 hours. Hence it covers 30 degrees ($360/12$) in one hour.

\implies In 20 minutes it covers 10 degrees ($30/3$). Hence the acute angle between hour hand and minute hand is 10 degrees.

53. Which number replaces the question mark?

$4, 7, 15, 31, ?, 19, 11, 6$

Answer:- (a)

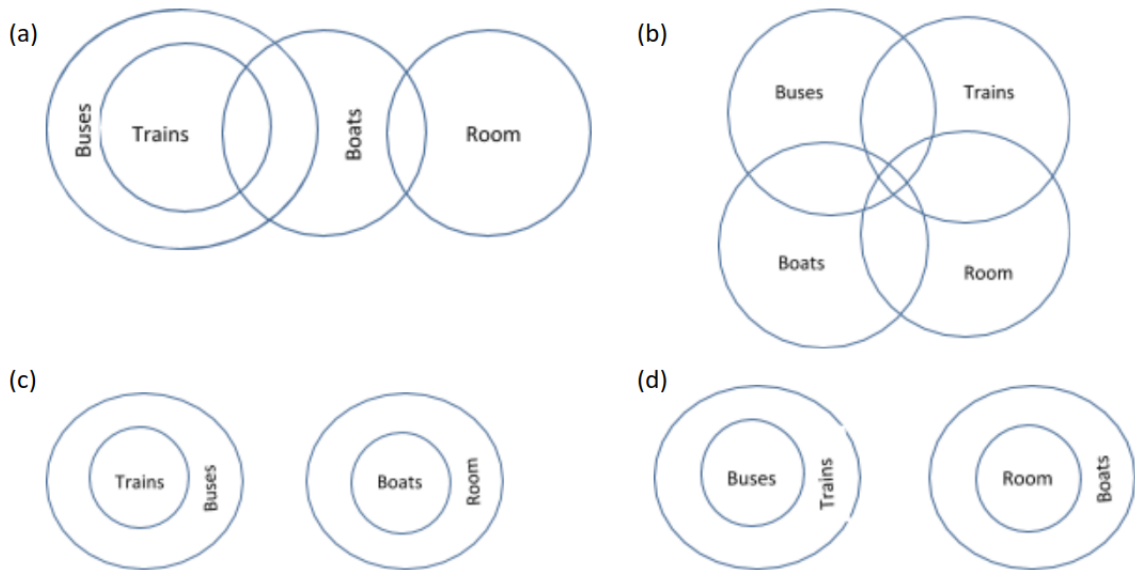
Starting on the extreme left and right hand of the row and working towards the centre, add the end digits together and subtract 3 to give the next value along to the left, and add 1 to give the next value along to the right. Repeat this sequence, working towards the centre.

$4 + 6 = 10 \implies 10 - 3 = 7, 10 + 1 = 11$

$7 + 11 = 18 \implies 18 - 3 = 15, 18 + 1 = 19$

$15 + 19 = 34 \implies 34 - 3 = 31, 34 + 1 = 35$

54. Analyze following diagrams and find out the diagram which accurately represents the given statement. STATEMENT: No room is trains, No boats are buses and all trains are buses



Answer:- (c)

DIRECTIONS FOR QUESTIONS 55 and 56: The capital letters in each of the following words are coded and written in small letters on the right side of each word, but the small letters do not appear in the same order as the letters in the word. Find out the codes for letters and answer the following questions:

KING : bdme
 RING : deob
 INK : emb
 IRK : oem

55. Which is the code for letter K?

Answer:- (b)

K-m, I-e, N-b, G-d, R-o

56. What would be the code (in correct order) for the word KIN?

Answer:- (d)

K-m, I-e, N-b, G-d, R-o

DIRECTIONS FOR QUESTIONS 57-60: Study the following information carefully and answer the questions given below it. Shyam, Raheem, Isaac, Hamza and Rohan help themselves to take some sweets from bowl. Four of them each take a gulab jamun. Raheem and Hamza do not take a burfi as all the other do. Infact Raheem takes only one sweet, which is a laddu. Apart from Raheem, only Shyam and Rohan do not take peda.

57. Who are the two people taking the same number and same type of sweets?

Answer:- (d)

Shyam - Burfi, Gulab Jamun

Raheem- Laddu

Isaac- Burfi, Gulab Jamun, Peda

Hamza- Gulab Jamun, Peda

Rohan- Burfi, Gulab Jamun

58. Who took three sweets?

Answer:- (a)

Shyam - Burfi, Gulab Jamun
Raheem- Laddu
Isaac- Burfi, Gulab Jamun, Peda
Hamza- Gulab Jamun, Peda
Rohan- Burfi, Gulab Jamun

59. Who only had peda and gulab Jamun?

Answer:- (d)

Shyam - Burfi, Gulab Jamun
Raheem- Laddu
Isaac- Burfi, Gulab Jamun, Peda
Hamza- Gulab Jamun, Peda
Rohan- Burfi, Gulab Jamun

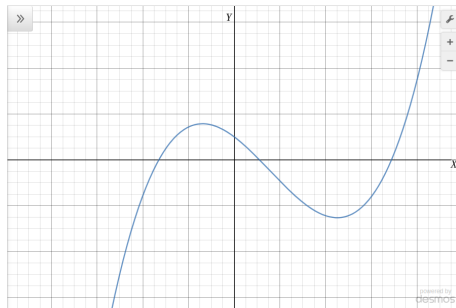
60. In total how many pieces of sweets were taken by the group?

Answer:- (c)

Shyam - Burfi, Gulab Jamun
Raheem- Laddu
Isaac- Burfi, Gulab Jamun, Peda
Hamza- Gulab Jamun, Peda
Rohan- Burfi, Gulab Jamun

3 Mathematics

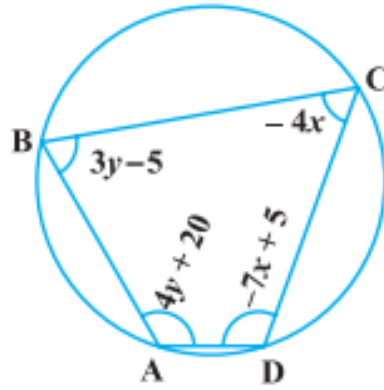
61. Following is the graph of $y=p(x)$, where $p(x)$ is a polynomial. The number of zeroes of $p(x)$ is



Answer:- (a)

Note that the curve crosses the X axis on 3 points. These are the points where $y = p(x)$ is zero. Hence number of zeroes is 3.

62. ABCD is a cyclic quadrilateral. Find the angle A of the cyclic



Answer:- (d)

$$\begin{aligned}
 \angle A + \angle B + \angle C + \angle D &= 360 \\
 -11x + 7y + 20 &= 360 \\
 -11x + 7y &= 340
 \end{aligned} \tag{1}$$

Also

$$\begin{aligned}
 \angle A + \angle C &= \angle B + \angle D = 180 \\
 -4x + 4y + 20 &= -7x + 3y \\
 3x + y &= -20
 \end{aligned} \tag{2}$$

Multiplying equation (1) and equation (2) with 3 and 11 respectively, we get

$$\begin{aligned}
 -33x + 21y &= 1020 \\
 33x + 7y &= -140
 \end{aligned}$$

Therefore $28y = 880$

Therefore $4y = 125$

63. The coordinates of the vertices A and B of the triangle ABC are $(2, 3)$, $(-2, 1)$ respectively. The coordinates of the centroid is $(1, \frac{2}{3})$. The coordinates of the vertex C are

Answer:- (d)

The

$$\frac{2 + (-2) + x}{3} = 1$$

Therefore $x = 3$

$$\frac{3 + 1 + y}{3} = \frac{2}{3}$$

Therefore $y = -2$

64. The value of $\cos x^\circ - \sin x^\circ$ ($0 \leq x < 45$) is

Answer:- (b)

$\cos x^\circ$ decreases from 1 to $\frac{1}{\sqrt{2}}$ as x increases from 0 to 45.

Hence $\cos x \geq \frac{1}{\sqrt{2}}$ in the given range of x .

Similarly $\sin x^\circ$ increases from 0 to $\frac{1}{\sqrt{2}}$ as x increases from 0 to 45.

Hence $\sin x \leq \frac{1}{\sqrt{2}}$ in the given range.

Therefore, $\cos x^\circ - \sin x^\circ > 0$ in the given range.

65. Which of the following is a false statement?

Answer:- (b)

Option (a) is true because, putting value of q as 0, 1, 2, ... will give us 1, 3, 5, ... respectively.

Option (b) is false. Since q is an integer it can be negative also. If we put $q = -1$, $4q + 1 = -3$ which is not positive.

Option (c) is true. When $q = 0, 1, 2, \dots$, we get 1, 3, 5, 7, 9, 11, 13, 15, 17, ...

Option (d) is true. -5 and -9 do not have a common factor other than 1.

66. In a deck of 52 cards, there are 4 suits (heart, diamond, spade, and club) of 13 cards each. Each suit has cards called ace, king, queen and jack, remaining 9 cards are numbered from 2 to 10. A card is drawn at random from a well-shuffled deck of 52 cards. The probability that the card is neither a heart nor a king is

Answer:- (a)

Since there are 4 kings and 13 hearts, in which 1 of the king is a heart as well, we have $13 + 3 = 16$ cards satisfying the negative condition Hence the prob of getting neither heart nor king is $1 - \frac{16}{52} =$

$$\frac{36}{52} = \frac{9}{13}$$

67. There is a circular path around the sports field. Maryam takes 20 minutes to drive one round of the field, while Ahmad takes 15 minutes for the same. Suppose they both start at the same point and the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer:- (a)

Here we need to find the LCM of 15 and 20 to find out the answer. Hence the answer is 60 min.

68. If $2^m - 2^{m-1} = 4$ then value of m^m is

Answer:- (b)

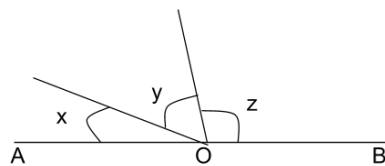
$$2^m(1 - 1/2) = 4$$

$$2^{m-1} = 4$$

$$m = 3$$

$$\text{Therefore } m^m = 27$$

69. In the given figure, AOB is a straight line. If $x : y : z = 4 : 5 : 6$, then $y =$



Answer:- (a)

Since AOB is a straight line. Sum of $x, y,$ and z will be 180° .

$$x + y + z = 180 \quad (3)$$

We also know $x : y : z = 4 : 5 : 6$. Hence we can assume $x = 4k, y = 5k, z = 6k$. From equation 3

$$\begin{aligned} x + y + z &= 180 \\ 4k + 5k + 6k &= 180 \\ 15k &= 180 \\ k &= 12 \end{aligned}$$

Hence $y = 5k = 5 \times 12 = 60^\circ$

70. The value of $\frac{\cos^2 \theta + \tan^2 \theta - 1}{\sin^2 \theta}$ is

Answer:- (c)

$$\begin{aligned} \frac{\cos^2 \theta + \tan^2 \theta - 1}{\sin^2 \theta} &= \frac{\cos^2 \theta + \tan^2 \theta - \cos^2 \theta - \sin^2 \theta}{\sin^2 \theta} \\ &= \frac{\tan^2 \theta - \sin^2 \theta}{\sin^2 \theta} \\ &= \frac{1}{\cos^2 \theta} - 1 = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \\ &= \sec^2 \theta - 1 = \tan^2 \theta \end{aligned}$$

71. A vertical pole of height 10 metres stands at one corner of a rectangular field. The angle of elevation of its top from the farthest corner is 30° , while that from another corner is 60° . The area (in m^2) of rectangular field is

Answer:- (a)

$$\begin{aligned} \tan 30^\circ &= \frac{10}{\text{diagonal}} \\ \text{or, } \frac{1}{\sqrt{3}} &= \frac{10}{\text{diagonal}} \\ \text{or, diagonal} &= 10\sqrt{3} \end{aligned}$$

$$\begin{aligned} \tan 60^\circ &= \frac{10}{\text{side1}} \\ \text{or, } \sqrt{3} &= \frac{10}{\text{side1}} \\ \text{or, side1} &= \frac{10}{\sqrt{3}} \end{aligned}$$

$$\text{diagonal}^2 = (\text{side1})^2 + (\text{side2})^2 \text{ (Pythagoras Theorem)}$$

$$\text{or, } 300 = \frac{100}{3} + (\text{side2})^2$$

$$\text{or, side2} = 20\sqrt{\frac{2}{3}}$$

$$\text{Therefore, Area of the rectangular field} = \text{side1} * \text{side2} = \frac{200\sqrt{2}}{3}$$

72. The mean weight of 150 students in a class is 60 kg. The mean weight of the boys is 70 kg while that of girls is 55 kg. Find the difference of number of boys and girls.

Answer:- (b)

Let bw and gw be the sum of weights of all boys and all girls respectively.

And let b and g be the number of boys and girls respectively. Mean wt of class = $\frac{bw + gw}{150}$

$$60 = \frac{bw + gw}{150}$$

$$bw + gw = 9000$$

$$\text{Mean wt of boys} = \frac{bw}{b}$$

$$70 = \frac{bw}{b}$$

$$bw = 70b$$

$$\text{Mean wt of girls} = \frac{gw}{g}$$

$$55 = \frac{gw}{g}$$

$$gw = 55g$$

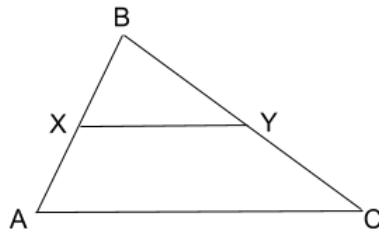
$$\text{Hence } 70b + 55g = 9000$$

We also know $b + g = 150$ (total no of students)

Hence solving two equations we get $b = 50$ and $g = 100$.

Hence difference of number of boys and girls is 50.

73. In the given figure, XY is parallel to AC and divides the triangle ABC into the two parts of equal area. Then the ratio $AX:AB$ equals



Answer:- (b)

Given,

$$\text{Area of } \triangle BXY = \text{Area of trapezium } AXYC$$

Hence,

$$\text{Area of } \triangle ABC = 2 \times \text{Area of } \triangle BXY$$

$$\implies \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BXY} = 2 \quad (1)$$

We know that the ratio of Area of similar triangles is equal to square of ratio of their sides. Here $\triangle ABC \sim \triangle BXY$

Hence

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BXY} = \left(\frac{AB}{BX}\right)^2 \quad (2)$$

From equations 1 and 2

$$\begin{aligned} \left(\frac{AB}{BX}\right)^2 &= 2 \\ \Rightarrow \frac{AB}{BX} &= \sqrt{2} \\ \Rightarrow \frac{AB}{AB - AX} &= \sqrt{2} \\ \Rightarrow \frac{AB - AX}{AB} &= \frac{1}{\sqrt{2}} \\ \Rightarrow 1 - \frac{AX}{AB} &= \frac{1}{\sqrt{2}} \\ \Rightarrow 1 - \frac{1}{\sqrt{2}} &= \frac{AX}{AB} \\ \Rightarrow \frac{AX}{AB} &= 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2} \end{aligned}$$

74. The value of k so that the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one root in common are

Answer:- (c)

From the second equation,

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\text{When } x = -1, 2 - k - 5 = 0, k = -3$$

$$\text{When } x = 4, 32 + 4k - 5 = 0, k = -27/4$$

75. Let us consider the following two arithmetic progressions with 100 elements in each

A: 5, 8, ...

B: 3, 7, ...

How many common elements are there in the AP sequences A and B?

Answer:- (d)

$$\text{A: } a = 5, d = 3; \text{ last element is } 5 + (100 - 1)3 = 302$$

$$\text{B: } a = 3, d = 4; \text{ last element is } 7 + (100 - 1)4 = 403$$

A: 5, 8, 11, 14, 17, 20, 23, ..., 302

B: 3, 7, 11, 15, 19, 23, 27, ..., 403

First common element = 11

Second common element = 23

The common elements 11, 23, ... are also in AP, where $a = 11$ and $d = 12$.

Let T_n be the last element of this AP which is less than 302. Hence,

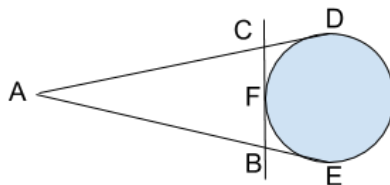
$$11 + (n - 1)12 \leq 302$$

$$\text{or, } n - 1 \leq 291/12$$

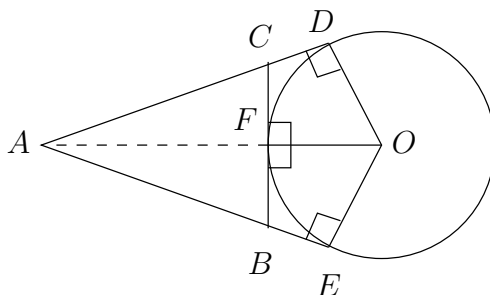
$$\text{or, } n \leq 25.25$$

But n is a natural number, the nearest integer which is lower than 25.25 is 25.

76. In the figure given below, AD, AE and BC are tangents to the circle at D, E, F respectively. Then



Answer:- (c)



In $\triangle AOD$,

$$AO^2 = AD^2 + DO^2 \tag{1}$$

In $\triangle AOE$,

$$AO^2 = AE^2 + EO^2 \tag{2}$$

Since $DO = EO =$ radius of the circle, from 1 and 2,

$$AD = AE \tag{3}$$

i.e; **Tangents from a single point to a circle will have the same length**

Similarly,

$$CD = CF, BE = BF \tag{4}$$

Now,

$$\begin{aligned} AD + AE &= (AC + CD) + (AB + BE) \\ \text{or, } 2AD &= AC + AB + (CD + BE), \quad \text{from 3} \\ \text{or, } 2AD &= AC + AB + (CF + BF), \quad \text{from 4} \\ \text{Therefore, } 2AD &= AC + AB + BC \end{aligned}$$

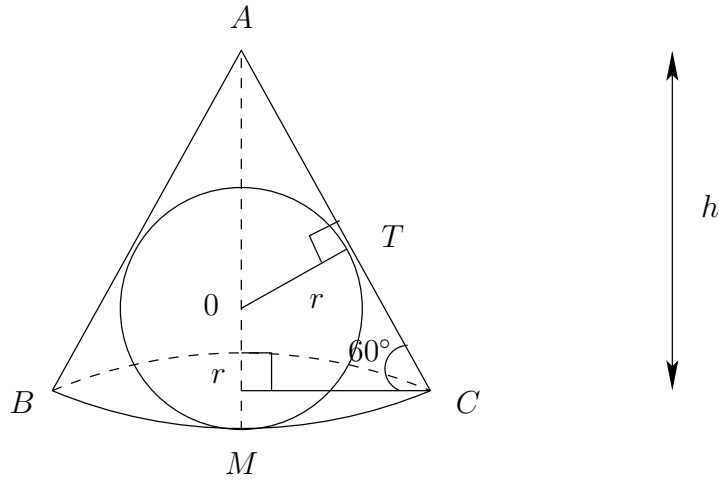
77. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is 0 then the product of the other two zeroes is

Answer:- (c)

In a cubic polynomial where α, β, γ are the zeros, the coefficient of x is $\alpha\beta + \beta\gamma + \alpha\gamma = b$
If any of them is 0, lets say γ , then the product of the other two, $\alpha\beta = b$

78. There is a right circular cone of height h and vertical angle 60° . A sphere when placed inside the cone, it touches the curved surface and the base of the cone. The volume of the sphere is

Answer:- (d)



Since $\angle ACM = 60^\circ$, $\angle CAM = 30^\circ$

In $\triangle AOT$,

$$\sin 30^\circ = \frac{OT}{OA} = \frac{r}{h-r}$$

or, $\frac{1}{2} = \frac{r}{h-r}$

or, $h-r = 2r$

or, $r = \frac{h}{3}$

$$\begin{aligned} \text{Volume of the sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{81}\pi h^3 \end{aligned}$$

79. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x+a$, then the value of a is

Answer:- (b)

$$\begin{array}{r} x^2 - 2x + k \) \ x^4 - 6x^3 + 16x^2 - 25x + 10 \\ \underline{x^4 - 2x^3 + kx^2} \\ -4x^3 - kx^2 + 16x^2 - 25x + 10 \\ \underline{-4x^3 + 8x^2 - 4kx} \\ (8-k)x^2 + (4k-25)x + 10 \\ \underline{(8-k)x^2 - 2(8-k)x + (8-k)k} \\ (2k-9)x + k^2 - 8k + 10 \end{array}$$

Now given, the remainder is $x + a$

Hence

$$x + a = (2k - 9)x + k^2 - 8k + 10$$

Now equating the coefficient of x and constant on both sides

$$2k - 9 = 1 \text{ and } k^2 - 8k + 10 = a$$

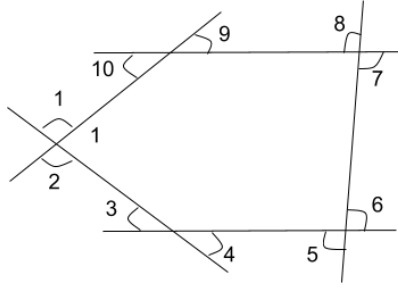
or, $k = 5$

Hence

$$5^2 - 8 \times 5 + 10 = a$$

or, $a = 25 - 40 + 10 = -5$

80. Find sum of the angles, $\sum_{i=1}^{10} (\angle i)$, in the diagram given below:



Answer:- (d)

$\angle 1 = \angle 2$ (vertically opposite angles)

$\angle 3 = \angle 4$ (vertically opposite angles)

$\angle 5 = \angle 6$ (vertically opposite angles)

$\angle 7 = \angle 8$ (vertically opposite angles)

$\angle 9 = \angle 10$ (vertically opposite angles)

$$2\angle 1 + 2\angle A = 360^\circ$$

$$\angle 1 = 180^\circ - \angle A$$

Similarly $\angle 3 = 180^\circ - \angle B$,

$$\angle 5 = 180^\circ - \angle C,$$

$$\angle 7 = 180^\circ - \angle D,$$

$$\angle 9 = 180^\circ - \angle E,$$

$$\begin{aligned} \text{Now } \sum_{i=1}^{10} (\angle i) &= 2(\angle 1 + \angle 3 + \angle 5 + \angle 7 + \angle 9) \\ &= 2(180 - \angle A + 180 - \angle B + 180 - \angle C + 180 - \angle D + 180 - \angle E) \\ &= 2(900 - 540) = 2 \times 360 = 720^\circ \end{aligned}$$